

**TITLE OF THE INVENTION****Method For Generating Many-Time Restrictive Blind Signatures**

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**RELATED APPLICATIONS:**

10 This application is based on U.S. Provisional Serial No. 60/161,062, filed October 25, 1999.

**BACKGROUND OF THE INVENTION**

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1. Field of the Invention

The invention disclosed broadly relates to cryptography and more particularly relates to digital signature methods.

In contrast to conventional digital signature schemes, blind signature schemes allow the recipient to obtain signatures for messages that the signer does not learn. If the recipient can get only one signed message from each execution of the signing operation by the signer, then the blind signature scheme is called *one-time*, otherwise it is called *many-time*. Many-time blind signatures have been used to build untraceable tickets, called *credentials*. Such tickets can be issued by one organization and verified by another. Each customer uses different pseudonyms with each organization and a ticket is simply a blind signature for a customer pseudonym. The blinding property allows one to use different pseudonyms for issuing and showing a ticket. Even if all organizations collude, they cannot trace which tickets belong to which customers. One-time blind signatures have been used to build practical offline and online untraceable electronic cash schemes, where the issuing organizations are banks, the recipients are merchants and the tickets can be used only once. Most electronic cash schemes based on blind

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signatures use the one-time form, mainly to avoid the problem of multiple copies of the same electronic coin.

For offline untraceable electronic cash, double spending of coins should be detectable after the fact, so that double spenders are identifiable if and only if they use a coin more than once. This problem has been addressed by using restrictive one-time blind signatures. The customer's identity is embedded into her pseudonyms in such a way that it is revealed if and only if she double spends. A general blind signature scheme would allow a customer to also obtain coins for pseudonyms of other customers or for pseudonyms that are not assigned to anyone. In contrast, restrictive blind signature schemes guarantee that customers form their pseudonyms in a way that preserves the customer's identity, which the signer has encoded into each issued pseudonym.

A related application area is untraceable membership cards, which can be stored in palmtops, smartcards, etc. Owners may use their membership cards online or offline, arbitrarily often, and in an untraceable way, i.e., several uses of the same card cannot be linked by the respective verifiers. However, issuers of membership cards require that membership cards can be used only by their owners, not by other individuals, even if the owners wish to lend their membership cards away. Purely cryptographic solutions to this problem cannot exist because whether a membership card is actually used by its owner or someone else, is not distinguishable by cryptographic means. It has been suggested to use a wallet-with-observer architecture, where every user has a personal device (wallet) that is in part controlled by an implanted tamper resistant security module (observer). The observers can be equipped with a biometric sensor which is a sufficiently powerful hardware basis for the problem at hand. The prior art relies heavily on the tamper resistance of observers, because if an attacker breaks his observer he can

not only lend his own membership cards to other individuals, but he can also forge new membership cards. Another approach relies on the tamper resistance of only observers with respect to transferability of membership cards. Attackers who break their observers can at most pool all the membership cards they already have, but cannot produce new ones. The approach includes a "cascade" signature scheme which has not been implemented.

What is needed in the prior art is a restrictive blind signature scheme that allows a recipient to obtain signatures for arbitrarily many (correctly formed) messages after only one interaction with the signer.

## SUMMARY OF THE INVENTION

A multiple use ticket generating method is disclosed which enables a recipient to obtain signatures for arbitrarily many (correctly formed) messages after only one interaction with the signer. The method provides a blind signature in a ticket, the signature having a multiple use with a built-in expiration. Then, the method develops a blinding value for the signature in a reproducible computation using a seed key substantially known only to the issuer of the ticket. The method implements a new class of man-time restrictive blind signature schemes almost as efficiently as do previous one-time restrictive blind signature methods.

The resulting ticket can be in the form of an electronic personal ticket, such as a season ticket for sporting events. Other forms for the ticket can include a personal license, such as a personal driver's license. The ticket has the property of being untraceable and has the advantage that the signature does not require an interactive signing protocol.

## **DESCRIPTION OF THE FIGURES**

Figure 1 shows a method for producing a signature.

Figure 2 shows a method for transforming a signature.

## **DESCRIPTION OF THE PREFERRED EMBODIMENTS**

An efficient implementation of a many-time restrictive blind signature scheme is disclosed. It uses no hash function, is about as efficient as previous one-time restrictive blind signature methods, and its security rests on a similar assumption as that of the ElGamal signature scheme. Applications for the new signature scheme are untraceable offline personal tickets, e.g., monthly season tickets, driver's licenses, or coupons that can be used multiple times until they expire. A computer system for carrying out the method of the invention is a standard general purpose data processor that includes a random access memory to store the program embodiment of the invention and a central processor to execute the instructions in the program embodiment. The computer system is connected to a network to generate and circulate untraceable tickets, licenses, or coupons that can be used multiple times until they expire.

### **Definitions**

A definition follows of many-time restrictive blind signatures. The formalization of restrictiveness follows ideas of Brands [B93], Franklin and Yung [FY93] and Pfitzmann and Sadeghi [PS99].

#### **Definition 1 (Many-time restrictive blind signature).**

A many-time restrictive blind signature scheme consists of a security parameter  $k \in \mathbb{N}$ , a signing key space  $X$ , a verifying key space  $Y$ , a message space  $M$ , a signature space  $\Sigma$ , a blinder space  $\Omega$ , a witness space  $W$ , and a relation  $make \subseteq M \times W$ . Also included is an equivalence

relation on  $W$  (equivalent witnesses  $v, w \in W$  are denoted  $v \equiv w$ ), more precisely, there are families of all these domains indexed by the security parameter  $k$ . If  $(w, m) \in \text{make}$  then we say that witness  $w$  makes message  $m$ . At system setup time, a particular security parameter is chosen and from then on, only one instance of each domain is used. Also included are two probabilistic protocol algorithms  $\text{gen}$ ,  $\text{sign}$ , a probabilistic protocol  $\text{trans}$  of two participants Bob and Verifier, and a deterministic algorithm  $\text{verify}$ , which are declared as follows:

$$\begin{aligned} (x, y) &\leftarrow \text{gen}(k) & \sigma &\leftarrow \text{sign}(x, m) \\ (m', \sigma') &\leftarrow \text{trans}(y, m, \sigma, w) & \text{acc} &\leftarrow \text{verify}(y, m, \sigma) \end{aligned}$$

All of them are efficiently computable. Given a security parameter  $k$ , the key generating algorithm  $\text{gen}$  returns a pair of a private signing key  $x \in X$  and a public verification key  $y \in Y$ . The algorithm  $\text{sign}$  takes as input a signing key  $x \in X$  and a message  $m \in M$ . It returns a signature  $\sigma \in \Sigma$ . The protocol  $\text{trans}$  takes as input for both Bob and the Verifier a verification key  $y$ , and only for Bob a message  $m$ , a signature  $\sigma$  and a blinder  $\omega$ . After the protocol, both Bob and the Verifier return the same message  $m'$  and signature  $\sigma'$ . The algorithm  $\text{verify}$  takes as input a public key  $y$ , a message  $m \in M$  and a signature  $\sigma \in \Sigma$  and returns a Boolean value  $\text{acc}$ . If  $\text{verify}(y, m, \sigma)$  returns True then the signature  $\sigma$  is called *valid* for  $m$  with respect to public key  $y$ , or the pair  $(m, \sigma)$  is *valid* for  $y$ .

**EFFECTIVENESS:** For every security parameter  $k$ , every key pair  $(x, y) \leftarrow \text{gen}(k)$ , and every message  $m \in M$  the algorithm  $\text{sign}(x, m)$  produces a valid signature  $\sigma$  for  $m$ . For all inputs as above, every blinder  $\omega \in \Omega$  and every signature  $\sigma \in \Sigma$  valid for  $m$  the algorithm  $\text{trans}(y, m, \sigma, \omega)$  returns a valid signature  $\sigma'$  for  $m'$ .

**RESTRICTIVENESS** with respect to  $\text{make}$  and  $\equiv$ : Every polynomial-time attacker who (i) obtains valid signatures  $\sigma_i$  ( $i = 0 \dots n$ ) from the signer for respective messages  $m_i$  of his

(adaptive) choice. The choice of each message he asks to be signed may depend on all messages previously chosen and the corresponding responses by the signer. The attacker also (ii) comes up with a new message  $m'$  and signature  $\sigma'$  and (iii) delivers  $n+1$  witnesses  $\omega_1, \omega_2, \dots, \omega_n, \omega'$  has only a negligible chance of achieving the following event: The signature  $\sigma'$  is valid for  $m'$ , the witnesses  $\omega_i, \omega'$  each match their messages  $m_i, m'$  and the witness  $\omega'$  is not equivalent to any of the witnesses  $\omega_i$  if any.

UNLINKABILITY: Let  $(m, \sigma), (m', \sigma')$  be two pairs valid with respect to  $y$ . Then for each internal choice  $r_V$  of the Verifier in *trans*, there is a unique blinder  $\omega \in \Omega$  and a unique internal choice, (i.e., a sequence of random bits used by a probabilistic algorithm)  $r_B$  for Bob in algorithm *trans*, such that the execution of *trans* ( $y, m, \sigma, \omega$ ) with internal choices  $r_B, r_V$  returns  $(m', \sigma')$ .

Note that previous one-time blind signature schemes use an interactive signing protocol from which the recipient gets a message and signature that he can later show to a verifier without interaction. Many-time blind signature schemes use a non-interactive signing protocol from which the recipient gets a message and signature that he can later transform and thereby show to many verifiers.

### 3 The Rust Signature Scheme

The proposed many-time blind signature scheme is referred to here as "RUST". The standard discrete log setting is adopted. Let  $p$  be a large prime,  $q$  be a large prime divisor of  $p-1$ . Typically,  $p$  and  $q$  will be chosen about 1024 bit and 160 bit long, respectively [O99].

Then  $\mathbb{Z}_p^*$  has a unique subgroup  $G_q$  of order  $q$  and since  $\mathbb{Z}_p^*$  is cyclic, so is  $G_q$ . Let  $g, g_1$  be generators of  $G_q$  that are chosen uniformly at random.

The private and public key spaces are  $ZZ_q$  and  $G_q$ , respectively. The message space is  $M = G_q^* \setminus \{1\}$ , where  $G_q^*$  is  $G_q$  except all members that disappear modulo  $q$ . The signature space is  $\Sigma = G_q^* \times ZZ_q \times ZZ_q^*$ , the space of blinders is  $\Omega = ZZ_q^*$ , the witness space is  $W = ZZ_q$ , the making relation is

$$5 \quad make = \{(m, \omega) \in M \times W \mid m = g_1^\omega \bmod p\},$$

and any two witnesses are equivalent, i.e.,  $\forall v, w \in W: v \equiv w$ . Key generation is by choosing a signing key  $x \in X$  uniformly at random and computing the corresponding verification key  $y = g^x \bmod p$ .

A signature  $(r, s, t) \in \Sigma$  (thus the name of the scheme) is *valid* for message  $m \in M$  with respect to public key  $y$  if the following *equation* holds:

$$10 \quad verify(y, m, \sigma) = g^{r+s} = y^{m+t} m^{rs} r^{rt} \bmod p. \quad (1)$$

A pair  $(m, \sigma)$  is called *terminated* if  $t = -m \bmod q$ , otherwise it is called *fresh*.

### 3.1 Producing Signatures

15 Let the generator  $g$ , and a key pair  $(x, y)$  be setup as above. A signature for a given message  $m \in M$  is constructed as shown in Figure 1:

One chooses  $a, b \in_R ZZ_q$  uniformly at random such that  $a+bm \neq -1 \pmod{q}$  in step (1) and computes the signature component  $r$  in step (2). If any of the values  $r, r-mx$  or  $(a+bm)rmx$  disappears modulo  $q$ , then the execution needs to be repeated from step (1). In step (3), the remaining signature values  $s, t$  are computed.

### 20 3.2 Transforming Signatures

Given a verification key  $y$  and a blinder  $\omega \in \Omega$ , a fresh pair  $(m, (r, s, t)) \in M \times \Sigma$  of a message and a signature is transformed into another pair  $(m', (r', s', t'))$ . The blinder  $\omega$  is required such that  $m^\omega \bmod p \neq 0 \pmod{q}$  (see Figure 2): In step (1) through (5) Bob forms the new message  $m'$  and the signature component  $r' = m^a r^b g^c y^d$  such that:

1. the exponents  $b = \frac{rt}{m+t}d$  and  $c = -\frac{a}{\omega m'} + d \frac{ms - \omega(r+s)m'}{\omega(m+t)m'} - \frac{1}{m'}$  are functions of  $a$  and  $d$ ,
2. the Verifier does not learn any information about Bob's input  $m, (r, s, t)$ ,
3. even if Bob deviated from the protocol, he could not end up with some  $r'$  for which he has a representation with respect to  $m, r, g$  only, i.e.,  $d = 0$ .

5 In detail, Bob chooses uniformly at random an auxiliary value  $\alpha \in_R \mathbb{Z}\mathbb{Z}q$ , and the Verifier chooses  $d \in_R \mathbb{Z}\mathbb{Z}q$  (step (1)). Then Bob computes the output message  $m' = m^\omega \bmod p$  in step (2). Bob further computes the auxiliary values  $(\beta, \gamma)$  and the preliminary signature component  $r^*$  in step (3). After sending  $m', r^*$  to the Verifier, he obtains in return the Verifier's choice  $d$  in order to compute the signature component  $r'$  in step (5). So does the Verifier. Only in the case if  $d$  or  $r'$  disappears modulo  $q$  must the protocol be repeated from step (1). Next, Bob computes the exponents  $a, b$  according to step (6) and the signature components  $s', t'$  according to step (7). He finally sends the signature components  $s', t'$  to the Verifier. Figure 2 illustrates transforming a signature.

15 *Remark 2.* The RUST signature scheme ensures that signers always produce fresh pairs of messages and signatures and that only fresh pairs can be transformed. Note that if  $t = -m \bmod q$  some quotients in *trans* were undefined. However, transformed pairs are always terminated, so that a Verifier cannot transform a pair further. This feature of RUST is not implied by restrictiveness (Definition 1).

20 The protocol *trans* can be made non-interactive if one is willing to rely on the obscurity of some hash function  $H$  as in the standard Fiat-Shamir technique [FS87]: Instead of sending  $m', r^*$  after step (3) and obtaining the Verifiers choice  $d$  in return, Bob can compute  $d = H(y, m', r^*)$  after step (3) by himself. After step (7), Bob then sends  $m', r^*, s', t'$  to the Verifier. Finally, the Verifier checks in addition to the verification equation (1) whether  $r' = (r^* y)^{H(y, m', r^*)} g^{-1/m'}$ .

25 The witness equivalence used for the RUST signature scheme is degenerate in the sense that any two witnesses are equivalent. This is no weakness of the RUST signature scheme, but allows producing and transforming signatures quite efficiently. Note that Brands suggests to use his one-time restrictive blind signature scheme for offline e-cash [B93] with the same



degenerate witness equivalence (and function *make*). In offline e-cash, the price for the increased performance is computational instead of unconditional non-frameability. For many-time restrictive blind signatures, like the RUST scheme, signer identification by (more than one) signatures is no issue, and thus framing of signers is no issue either.

#### 5 4. Main Result

In order to analyze the security of a proposed many-time restrictive blind signature scheme, referred to here as RUST, one needs the following two assumptions. These assumptions are not among the intensely investigated complexity theoretic assumptions like the discrete logarithm assumption [MOV97]. Nevertheless, they also underlie for example the ElGamal signature scheme and its derivatives without having been made explicit in previous work.

##### Assumption 1.

For some natural number  $n \in \mathbb{N}$ , let  $g_i$  ( $i \in [1, n]$ ) be generators of  $G_q$ , and define the function

$$F_{g_1, g_2, \dots, g_n}(x_1, x_2, \dots, x_n) = \prod_{i=1}^n g_i^{x_i} \mod p$$

that takes arguments  $x = (x_1, x_2, \dots, x_n) \in \mathbb{Z}_q^n \setminus \{(0, 0, \dots, 0)\}$ . Then the function

$$F_{g_1, g_2, \dots, g_n}(x) \mod q$$

is an implementation of a random oracle [BR93]. (Note the difference of the moduli  $p$  and  $q$ !)

##### Assumption 2.

20 If at all, a polynomial-time attacker  $A$  can compute valid pairs of messages and signatures with respect to a given verification key  $y$ , but then only as follows:

- First pick a set of  $n \geq 1$  generators  $h_1, \dots, h_n$  of  $G_q$ ,
- choose tuples  $\mathbf{a}, \mathbf{b} \in \mathbb{Z}_q^n$ ,
- form the message  $m' = F_{h_1, \dots, h_n}(\mathbf{a})$  and the signature component  $r' = F_{h_1, \dots, h_n}(\mathbf{b})$ ,
- 25 – and finally compute the signature components  $s', t'$ .

Without loss of generality, the attacker can be assumed to pick the generators  $h_1, \dots, h_n$  such that he cannot feasibly find a representation of 1 with respect to  $h_1, \dots, h_n$  in  $G_q$ . Otherwise, he could represent at least one of the generators with respect to the others, and thus he could pick a proper subset of  $\{h_1, \dots, h_n\}$  in the first step above, adapt the following steps accordingly and end up with the same result  $(m', (r', s', t'))$ .

A similar assumption has been used to reason about the security of ElGamal signatures [EG85], but those assumptions were left implicit.

**Theorem 3.** *Under assumptions A1 and A2, RUST is a many-time restrictive blind signature scheme.*

*Proof.* Check effectiveness, restrictiveness and blindness in turn.

**Effectiveness of sign:** Under Assumption A1, the probability to make a choice  $a, b \in \mathbb{Z}_q$  such that any of the values  $r, r - mx$  or  $D \stackrel{\text{def}}{=} (a + bm)r + mx$  disappears modulo  $q$  is negligible and so is the probability to repeat step (2) of algorithm *sign*. In order to verify algorithm *sign* (see Figure 1), insert its output into the right hand side of verification equation (1):

$$\begin{aligned}
 y^{m+t} m^{ms} r^{rt} &= g^{x(m+t)} m^{ms} (m^a g^b)^{rt} \\
 &= g^{x(m+t) \frac{r-mx}{D}} m^{\frac{amr}{D}(mx-r)} m^{\frac{amr}{D}(r-mx)} g^{\frac{bmr}{D}(r-mx)} \\
 &= g^{\frac{mx}{D}((a+bm)r+mx+r-mx)} g^{\frac{bmr}{D}(r-mx)} \\
 &= g^{\frac{mrx}{D}(a+bm+1) + \frac{mr}{D}(br-bmx)} \\
 &= g^{\frac{mr}{D}(ax+x+br)} \\
 &= g^{\frac{r}{D}(ar+amx+mx+bmr-ar)} \\
 &= g^{\frac{r}{D}(D+amx-ar)} \\
 &= g^{r + \frac{ar}{D}(mx-r)} \\
 &= g^{r+s} \pmod{p} .
 \end{aligned}$$

The signer produces fresh signatures because he chooses

$$t = m \frac{r - mx}{(a + bm)r + mx} \neq m(-1) = -m \pmod{q} \text{ according to the condition } a \neq bm + 1 \pmod{q} \text{ in}$$

step (1) of Figure 1. The signature components  $r$  and  $t$  do not disappear modulo  $q$  because of the loop condition in step (2).

- 5 **Effectiveness of trans:** The following verification is prepared by expressing Bob's signature components  $r'$  and  $s'$  in terms of Bobs input and his internal choices  $\alpha, d$  and by using the definitions of  $\beta$  and  $\gamma$  according to step (3) of Figure 2:

$$r' = (r * y)^d g^{\frac{-1}{m'}} = (m^\alpha r^\beta g^\gamma y)^d g^{\frac{-1}{m'}} = m^{\alpha d} r^{\frac{rt}{m+r}d} g^{\frac{ms - \omega(r+s)m'}{\omega(m+t)m'}d - \frac{\alpha d}{\omega m'} - \frac{1}{m'}} y^d \quad (2)$$

$$s' = \frac{art - bms}{\omega rt} r' = \frac{r'}{\omega rt} \left( \alpha d r t - \frac{rt}{m+t} d m s \right) = d \frac{r'}{\omega} \left( \alpha - \frac{ms}{m+t} \right) . \quad (3)$$

Under Assumption A1, the probability of choosing  $a \in \mathbb{Z}\mathbb{Z}_q, d \in \mathbb{Z}\mathbb{Z}_q^*$ , such that  $r' = 0 \pmod{q}$  is negligible, and so is the probability of repeating after step (5). Next, insert the output  $m', (r', s', t')$  of algorithm *trans* into the verification equation (1) and by inserting the expressions for  $m' = m^\omega$  and  $r'$  according to equation (2):

$$\begin{aligned}
 & y^{m'+t'} m'^{m's'} r'^{r't'} \\
 &= m^{\frac{\omega m' (art - bms)}{\omega rt}} (m^{\alpha d} r^{\frac{rt}{m+t} d} g^{\frac{ms - \omega(r+s)m'}{\omega(m+t)m'} d - \frac{\alpha d}{\omega m'} - \frac{1}{m'}} y^d)^{-r'm'} \\
 &= m^{\frac{\omega m' r'}{\omega} (ad - \frac{dms}{m+t})} (m^{\alpha d} r^{\frac{rt}{m+t} d} g^{\frac{-\omega(r+s)m'}{\omega(m+t)m'} d} y^d)^{-m'r'} g^{(\frac{ms}{\omega(m+t)m'} d - \frac{\alpha d}{\omega m'} - \frac{1}{m'})(-m'r')} \\
 &= m^{(\alpha d - \frac{dms}{m+y})m'r'} (m^{\alpha d} r^{\frac{rt}{m+t} d} g^{\frac{r+s}{m+t} d} y^d)^{-m'r'} g^{(\frac{\alpha d}{\omega} - \frac{dms}{\omega(m+t)} + 1)r'} \\
 &= \underbrace{(m^{ms} r^{rt} g^{-(r+s)} y^{m+t})}_{=1}^{-d \frac{m'r'}{m+t}} g^{r' + \frac{dr'}{\omega} (\alpha - \frac{ms}{m+t})} = g^{r'+s''} \pmod{p} .
 \end{aligned}$$

For the final rewriting use the expression (3) for  $s'$ . According to step (7) of Figure 2, Bob produces terminated pairs because  $t' = -m'$ . This guarantees  $t' \neq 0 \pmod{q}$  because  $m'$  is presumed not to disappear modulo  $q$ . The signature component  $r'$  does not disappear modulo  $q$  because of the loop condition in step (5).

**Restrictiveness:** First consider private key related attacks. Consider a polynomial-time attacker who has obtained  $n \in \mathbb{IN}$  valid pairs  $(m_i, (r_i, s_i, t_i))$  of messages and signatures for  $i = 1, \dots, n$  from the signer. The signer has chosen  $r_i = m_i^{a_i} g^{b_i} \pmod{p}$ , and has computed the signature components  $s_i, t_i$  according to Figure 1. The signature components  $r_i$  release no information about the choices  $a_i, b_i$  to a polynomial-time attacker, so we need to look only at  $s_i$  and  $t_i$ .

According to Figure 1,  $s_i = \frac{a_i r_i}{m_i} t_i$ , which reveals the signer's choices  $a_i$ . From the  $t_i$ , the attacker learns the following system (4) of  $n$  linear equations over  $\mathbb{ZZ}_q$  in  $n+1$  variables, namely  $b_i, x$  for  $i = 1, \dots, n$ :

$$m_i(r_i - m_i x) = t_i(a_i r_i + b_i m_i r_i + m_i x) \quad (4)$$

$$\Leftrightarrow b_i m_i r_i t_i + m_i x(t_i + m_i) = m_i r_i - a_i r_i t_i .$$

The values  $x$  and  $b_i$  are undetermined because  $t_i, m_i \neq 0$ , and therefore valid signatures release no more information about  $x = \log_g y$  to a polynomial-time attacker, than  $y$  itself.

5        Next, show that an attacker who has not received any valid RUST signature with respect to a public key  $y$  cannot feasibly fabricate a valid signature for any message on his own (Case 0). An attacker who has got valid signatures for one or more messages  $m_i$  is considered afterwards (Case 1).

10        **Case 0:** By contradiction to restrictiveness (Definition 1), assume an attacker who has no valid pairs of messages and signatures in the first place ( $n = 0$  in Definition 1), but succeeds to come up with a message  $m$  for which he has a witness  $\omega \in \Omega$  that makes  $m$ , i.e.,  $m = g_1^\omega$ , and a valid signature  $\sigma$ . (For lack of input pairs to *trans*, plain identifiers are used for the outputs, i.e., no primes.) According to Assumption A2, the attacker uses 3 parameters  $a, c, d \in \mathbb{Z}\mathbb{Z}_q$  in order to build the signature component:

$$r = m^{a'} g^{c'} y^{d'} \bmod p .$$

15        Because  $m$  must be chosen to be  $g_1^\omega$ , the only elements of  $G_q$  an attacker might use successfully to build  $r$  are those occurring in the verification equation (1), namely  $m, g, y$ . Would he use any other independently chosen element  $h \in G_q$  and succeed to find a valid  
20        signature, then the verification equation would reveal a representation of  $h$  with respect to  $m, g, y$ , which contradicts the discrete logarithm assumption.

Inserting the expression for  $r$  into the verification equation (1) yields:

$$g^{r+s} = y^{m+t} m^{ms} r^{rt} = y^{m+t} m^{ms} (m^{a'} g^{c'} y^{d'})^{rt} = y^{m+t} g_1^{\omega ms} (g_1^{\omega a'} g^{c'} y^{d'})^{rt} ,$$

which can be rewritten as:

$$g^{r+s-c'rt} = y^{m+t+d'rt} g_1^{\omega(ns+a'rt)}.$$

Since the bases  $g$ ,  $y$  and  $g_1$  are chosen independently, the only feasible way for the attacker to solve (5) is by letting the exponents of  $g$ ,  $y$  and  $g_1$  disappear.

5 This leads to the following linear system (6) of 3 equations in 2 variables  $s$  and  $t$ , over  $\mathbb{Z}\mathbb{Z}_q$ :

$$\begin{aligned} \omega ms + \omega a'rt &= 0 \\ -s + c'rt &= r \\ + (d'r + 1)t &= -m \end{aligned} \quad (6)$$

This system can be solvable only if the corresponding  $3 \times 3$  determinant disappears:

$$\begin{vmatrix} \omega m & \omega a'r & 0 \\ -1 & c'r & r \\ 0 & d'r + 1 & -m \end{vmatrix} = ((1 + a' + c'm) + d'r)\omega mr = 0 \quad (7)$$

Since neither  $\omega$  nor  $m$  nor  $r$  may disappear modulo  $q$ , this condition (7) can be met only if  $(1 + a' + c'm) + d'r = 0$ . Here, the factors  $m$  and  $r$  are determined only after  $\omega$  respective  $a'$ ,  $c'$ ,  $d'$  have been chosen, and by Assumption A1, neither  $m$  nor  $r$  can be predicted or coerced to any particular value. Hence the only way to let the determinant (7) disappear is to let  $1 + a' + c'm = d' = 0 \pmod{q}$ . However, protocol *trans* ensures with overwhelming probability that a dishonest Bob ends up with a representation of  $r$  whose exponent  $d'$  of  $y$  is not disappearing regardless of how Bob chooses  $r^*$ . Note that Bob must provide  $r^*$  *before* the Verifier sends his  $d$  and forms the signature component  $r = (r^*y)^d g^{-1/m}$  in step (5).

20 **Case 1:** Due to the degenerate equivalence  $\equiv$  of witnesses, i.e., any two witnesses are equivalent, restrictiveness is satisfied whenever the attacker has obtained at least one valid pair  $(m, \sigma)$  and comes up with a new pair  $(m', \sigma')$  and a witness making  $m'$ . Restrictiveness requires no more, and thus nothing needs to be shown.

25 **Blindness:** Show that for each fresh valid pair  $(m, \sigma)$ , where  $t \neq -m \pmod{q}$ , and each terminated valid pair  $(m', \sigma')$ , where  $t' = -m' \pmod{q}$ , of messages and RUST signatures, and each choice  $d \in \mathbb{Z}\mathbb{Z}_q^*$  of the verifier in *trans*, there is exactly one input  $\omega \in \mathbb{Z}\mathbb{Z}_q^*$  and one value

$\alpha \in \mathbb{Z}\mathbb{Z}_q^*$  such that *trans* maps  $(m, \sigma)$  to  $(m', \sigma')$ . (Note that the value  $r^*$  of the verifier's view on Bob in *trans* is a one-to-one map of the other elements  $d, m', r'$  of his view, and thus from an information theoretic viewpoint, it suffices to consider  $d, m', \sigma'$  as the verifier's view.) In the following, all steps refer to protocol *trans* in Figure 2.

- 5 First show there is at most one pair  $(\alpha, \omega)$ : It is immediate from step (2) that the blinder  $\omega = \log_m m'$  is uniquely determined. Furthermore, for each  $d \in \mathbb{Z}\mathbb{Z}_q^*$  is obtained from steps (7), (6) and (3) in turn the following expression for  $s'$ :

$$s' = \frac{art - bms}{\omega rt} r' = \frac{\alpha drt - \beta dms}{\omega rt} r' = \frac{\alpha rt - \frac{rt}{m+t} ms}{\omega drt} r' \pmod{q} .$$

10 Since all  $r, t, d, r', (m+t)$  are presumed not to disappear modulo  $q$ , the internal choice  $\alpha$  of Bob is uniquely determined as follows:

$$\alpha = \frac{\omega s'}{dr'} + \frac{ms}{m+t} \pmod{q} . \quad (8)$$

15 Next show that the uniquely determined pair  $(\alpha, \omega)$  from above transforms a fresh valid pair  $(m, \sigma)$  of message and signature into a terminated valid pair  $(m', \sigma')$ . Since  $(m, \sigma) = (m, (r, s, t))$  is presumed a fresh valid pair, we can rewrite the verification equation (1) for  $(m, (r, s, t))$  as follows:

$$\begin{aligned} g^{r+s} &= pk^{m+t} m^{ms} r^{rt} \\ \Leftrightarrow r^{rt} &= g^{r+s} pk^{-(m+t)} m^{-ms} , \text{ where } t \neq -m \pmod{q} . \end{aligned} \quad (9)$$

- 20 Furthermore, the unique  $\alpha$  in equation (8) also determines a unique  $\Upsilon$  in step (3), namely:

$$\begin{aligned}\gamma &= \frac{ms - \omega(r+s)m'}{\omega(m+t)m'} - \frac{\alpha}{\omega m'} = \frac{ms - \omega(r+s)m'}{\omega(m+t)m'} - \left( \frac{s'}{dm'r'} + \frac{ms}{\omega m'(m+t)} \right) \\ &= -\left( \frac{r+s}{m+t} + \frac{s'}{dm'r'} \right) \pmod{q} .\end{aligned}$$

Next, evaluate  $r'$  according to step (3) by inserting  $r^t$  from equation (9),  $\alpha$  from equation (8),  $\beta = rt/(m+t)$  from step (3) and  $\gamma$  from equation (10):

$$\begin{aligned}r' &= (r * pk)^d g^{\frac{1}{m'}} \\ &= (m^\alpha r^\beta g^\gamma pk)^d g^{\frac{1}{m'}} \\ &= m^{\alpha d} r^{\frac{rt}{m+t}d} g^{\gamma d \frac{1}{m'}} pk^d \\ &= m^{\alpha d} (g^{r+s} pk^{-(m+t)} m^{-ms})^{\frac{d}{m+t}} g^{\gamma d \frac{1}{m'}} pk^d \\ &= m^{(\frac{\omega s'}{dr'} + \frac{ms}{m+t})d} (g^{r+s} m^{-ms})^{\frac{d}{m+t}} g^{-(\frac{r+s}{m+t} + \frac{s'}{dm'r'})d - \frac{1}{m'}} \\ &= m^{\frac{\omega s'}{r'} - \frac{s'}{m'r'} - \frac{1}{m'}} \\ &= m^{\frac{s'}{r'} - \frac{s'+r'}{m'r'}} \pmod{p} .\end{aligned}\tag{11}$$

- 5 Finally, check that the values  $m', r', s', t'$  satisfy the verification equation (1) if  $r'$  is inserted from (11) and use  $t' = -m' \bmod q$  from step (7):



$$pk^{m'+t'} m'^{m's'} r'^{r't'} = pk^{m'-m'} m'^{m's'} (m'^{\frac{s'}{r'}} g^{-\frac{s'+r'}{m'r'}})^{-r'm'} = g^{r'+s'} \pmod{p} .$$

This concludes the proof.

A restrictive blind signature scheme has been presented that allows a recipient to obtain signatures for arbitrarily many (correctly formed) messages after only one interaction with the signer. Signing, transforming and verifying costs two, six, and six full length modular exponentiations, respectively. For transforming and verifying, count the exponentiations of Bob and of the Verifier in *trans*, respectively. This compares to two, five and four modular exponentiations of the signer and recipient during the signing protocol and verification of the one-time restrictive blind signature protocol proposed by Chaum, Pedersen [CP92] and later by Brands [B93].

Various illustrative examples of the invention have been described in detail. In addition, however, many modifications and changes can be made to these examples without departing from the nature and spirit of the invention.

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